# A Practitioners Toolkit on Valuation

Part I: (Un)Levering the Cost of Equity and Financing Policy with Constant Expected Free Cash Flows: APV, WACC and CFE

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## 1 Introduction

Valuation is at the heart of Finance, and it is by now well understood that the discounted cash flow (DCF) analysis provides the best framework to value projects and companies. When valuing companies in a DCF-framework, three methods are commonly applied: Adjusted Present Value (APV), the Weighted Average Cost of Capital (WACC), and valuation of the Cash Flow to Equity (CFE). Each of these methods at some point requires to lever or unlever the cost of equity. When using the APV, the starting point is to value the Free Cash Fow at the unlevered cost of capital or equity, which results in the unlevered value ($V_U$) of the firm. If the firm is financed with equity ($E$) as well as debt ($D$), the debt induces a tax shield ($TS$) the value of which is added to the unlevered firm value to obtain total firm value ($V$). When using the WACC, the Free Cash Flow is valued at the weighted averaged cost of equity and debt (after tax), yielding total firm value. The WACC requires as one of its inputs a levered cost of equity. Finally, when using the Cash Flow to Equity approach, one considers the cash flow that accrues to the shareholders only, and discounts this at the levered cost of equity. Combining the resulting equity value ($E$) with the current level of debt ($D$) yields total firm value ($V$) again.

Pracitioners as well as standard text books commonly use a specific way of (un)levering the cost of equity. In particular, two different ways of relating the unlevered cost of equity ($k_U$) to the levered cost of equity ($k_E$) are commonly used:

$$k_E = k_U + \frac{D}{E}(1 - T_C)(k_U - k_D), \qquad (1)$$

$$k_E = k_U + \frac{D}{E}(k_U - k_D). \qquad (2)$$

Here $k_D$ is the cost of debt for the firm, and $T_C$ is the corporate tax rate. Although most text books motivate the difference between these two methods by the presence or absence of corporate taxes ($T_C$), the claim in this article is that both methods are correct, also in the presence of corporate taxes, but apply to different assumptions on the financing policy of the firm. The use of the different methods, APV, WACC and CFE, are often thought to lead to different valuation results. Below we will show though, that the three different methods are entirely consistent with each other, provided that the cost of equity is (un)levered in the right way. Specifically, our main result is that if the financing policy is to have a constant level of debt, Equation (1) applies, whereas Equation (2) applies if the policy is to have a constant Debt/Equity ratio. In this article we will assume that there is no growth in the Free Cash Flow, and our exposition is closely related to Stanton & Seasholes (2005). In Part II of this article we will analyze the same problem for a firm with (constantly) growing Free Cash Flows and show that a similar issue of un(levering) the cost of equity applies. There also, unlike common wisdom, we will show that APV, WACC and CFE are entirely consistent, provided that the cost of equity is (un)levered in the right way.[1]

In this first part, we will show how APV, WACC

[1] Our results are not new - useful references are Modigliani & Miller (1963), Myers (1974), Miles & Ezzell (1980), Harris & Pringle (1985), Lewellen & Douglas (1986), Ruback (2002), and Stanton & Seasholes (2005). However, these results do not show up in commonly used text books.

and CFE are related under two different financial policies: a constant level of debt and a constant Debt/Equity ratio of the firm. Depending on the financing policy of the firm, the (un)levering should be done either using (1) or (2). The resulting valuations with the three methods will be the same. The analysis will be done assuming a constant expected Free Cash Flow. In Part II of this article, we will show how the analysis can be extended in case the Free Cash Flow is growing at a constant rate.

## 2 The case of constant debt

We will first show the equivalence of the three approaches for a firm that has a Free Cash Flow (FCF) dat is expected to remain constant and not to grow. The firm has a given amount of debt $D$, and its financing policy is that this level of debt will not change. We will illustrate the results with a simple example, for which the inputs are given in Exhibit 1.

| Exhibit 1: Input data | | | |
|---|---|---|---|
| Rf | 4.0% | FCF | 200.00 |
| Rm-Rf | 5.0% | kd*D | 50.00 |
| Bu | 0.80 | kD*D*Tc | 15.00 |
| ku | 8.0% | Change D | - |
| kD | 5.0% | CFE | 165.00 |
| Tc | 30% | | |
| | | | |
| D | 1000 | | |

### 2.1 Adjusted Present Value

With a constant expected FCF the all-equity or unlevered value of the firm is calculated as

$$V_U = \frac{FCF}{k_U}. \tag{3}$$

In our numerical example, unlevered beta is 0.80, the risk free rate is 4.0% and the market risk premium is 5.0%, yielding an unlevered cost of capital $k_U =$ 8.0%. With an expected FCF of 200, the unlevered firm value equals $V_U = 200/8\% = 2{,}500$.

If there would be no debt, the equity value would be equal to the unlevered firm value. If there is debt, as in our example, this yields tax savings on the interest payments which gives rise to a tax shield, which is added to the unlevered value of the firm to obtain total firm value. This is the Adjusted Present Value method:

$$V = V_U + TS. \tag{4}$$

To determine the value of the tax shield ($TS$), note that the annual tax saving is $k_D \times D \times T_C$. Since the level of debt is constant, the tax savings are linked directly to the interest payments $k_D \times D$ and have the same risk profile as the debt. There is a risk that the tax savings will not be realized, which is essentially the risk that the firm cannot pay the interest rate and therefore defaults on its debt. At the same time the maximum annual tax saving is always $k_D \times D \times T_C$ just like the maximum interest payment (if there is no default) equals $k_D \times D$. The risk of default is incorporated in the cost of debt $k_D$, which in general will exceed the risk free interest rate ($R_f$). We therefore claim that (following Myers (....)) the relevant discount rate for the tax shield is $k_D$. With a constant (eternal) debt, and therefore tax savings, this leads to a simple expression for the tax shield:[2]:

$$TS = \frac{k_D \times D \times T_C}{k_D} = D \times T_C. \tag{5}$$

In our example, the debt is 1000 and the tax rate 30%, yielding a tax shield of 300. Totoal company value using the Adjusted Present Value therefore equals 2,800, which consists of 1000 Debt and 1,800 equity. This is summarized in Exhibit 2.

[2] This formula is easily adjusted ot the case where is not eternal and not constant, as long as the changes in the level of debt are know a priori.

| Exhibit 2: Firm Value using the Adjusted Present Value Constant Debt | | | |
|---|---|---|---|
| Vu | 2,500.00 | E | 1,800.00 |
| TS | 300.00 | D | 1,000.00 |
| V | 2,800.00 | V | 2,800.00 |

## 2.2 Weighted Average Cost of Capital

The Weighted Average Cost of Capital (WACC) is the discount rate that, when applied to the Free Cash Flow, immediately yields total firm value (i.e., including the tax shield):

$$V = V_U + TS = \frac{FCF}{WACC}, \quad (6)$$

$$WACC = \frac{E}{V} k_E + \frac{D}{V} (1 - T_C) k_D. \quad (7)$$

The main issue when applying the WACC is which cost of equity to apply, as this is now the cost of *levered* equity. In order to derive this cost of equity, the starting point is that the left hand side and the right hand side of the balance sheet as in Exhibit 2, must give the same dollar (required) return:

$$k_U \times V_U + k_D \times TS = k_E \times E + k_D \times D.$$

Manipulating this equality, and using that the tax shield is $TS = D \times T_C$, the resulting cost of equity is

$$k_E = k_U + \frac{D}{E} (1 - T_C) (k_U - k_D),$$

which is Equation (1) above. Using this cost of equity in the WACC in Equation (7) gives the same total firm value as in the Adjusted Present Value methodology.

In our example, the levered cost of equity in (1) and the WACC in (7) are

$$k_E = 8.0\% + \frac{1,000}{1,800} \times 70\% \times (8\% - 5\%)$$
$$= 9.2\%$$
$$WACC = \frac{1,800}{2,800} \times 9.2\% + \frac{1,000}{2,800} \times 70\% \times 5\%$$
$$= 7.1\%.$$

Applying this WACC to the Free Cash Flow of 200, the resulting firm value is 200/7.1% = 2,800 as in the Adjusted Present Value calculation in Exhibit 2.

Note that by substituting (1) into (7), we can also write the WACC as:

$$WACC = \left(1 - \frac{D \times T_C}{V}\right) k_U. \quad (8)$$

The interesting thing to note here is that the WACC in this setting does not in any way depend on the cost of debt $k_D$, only on the unlevered cost of capital $k_U$ and the financial leverage of the company as measued by $D \times T_C / V$.

## 2.3 Cash Flow to Equity

The third valuation method is to value equity directly, by discounting the cash flow that accrues to the shareholders, the Cash Flow to Equity (CFE), and discouning it at the cost of equity. Adding the level of debt to this equity, gives total firm value. The cash flow that accrues to the shareholders, is the Free Cash Flow, minus the cost of debt ($k_D \times D$), plus the tax saving ($k_D \times D \times T_C$) plus the net increase in debt ($\triangle D$). Thus, a repayment of debt leads to a lower CFE, whereas new debt increases the CFE. The Cash Flow to Equity is thus calculated as:

$$CFE = FCF - k_D (1 - T_C) D + \triangle D. \quad (9)$$

In the case of no growth and constant debt, the change in debt, $\triangle D$, equals zero. We can then value the equity by discounting the CFE at the (levered) cost of equity in (1), that is also present in the WACC. After all, this is the cost of equity that reflects all risks that are present in equity.

In our example, the Cash Flow to Equity is

$$CFE = 200 - 5\% \times 70\% \times 1000 = 165.$$

Discounting this at the cost of equity gives 165/9.2% = 1,800 as in Exhibit 2. This illustrates that in our simple example the three valuation approaches indeed lead to the same value of equity as well as total firm value.

## 3 The case of a constant Debt/Equity ratio

Using the same firm that has a constant expected Free Cash Flow, we now show how the cost of equity needs to be (un)levered when it is not the level of debt that is constant, but when the Debt/Equity ratio is constant. Thus, the financing policy of the firm is such that it continuously rebalances its debt and equity, so that the Debt/Equity ratio ($D/E$ ) does not change. As in the previous section we consider each of the three valuation approaches, APV, WACC, and CFE, and show how they lead to the same valuation.

### 3.1 Adjusted Present Value

When applying the Adjusted Present Value method, the starting point is again the unlevered value of the firm as in Equation (3), to which the value of the tax shield is added as in Equation (4). The key difference is the valuation of the tax shield ($TS$). As in the case of constant debt, the tax saving at the outset is $k_D \times D \times T_C$. The key difference is now that the level of debt ($D$) will be adjusted continuously in order to keep the Debt/Equity ratio fixed. Since the left hand side of the balance sheet is determined by both $V_U$ and $TS$ and the right hand side by $E$ and $D$, the only way to keep $D/E$ constant is by keeping $TS/V_U$ constant. Thus, we need to adjust the tax shield $TS$ according to changes in $V_U$, implying that the risk in the tax shield now mirrors the risk in the unlevered firm value $V_U$. Unlike the case of constant debt, this means that the risk in the tax shield now equals the risk in the unlevered firm, and the relevant discount rate for the the tax shield is $k_U$. Thus,

$$TS = \frac{k_D \times D \times T_C}{k_U}. \tag{10}$$

In our example, this means that the value of the tax shield equals

$$TS = \frac{5\% \times 1000 \times 30\%}{8\%} = 187.5.$$

Total company value using the APV therefore equals 2,500 + 187.5 = 2,687.5, consisting of 1,000 debt and 1,687.5 equity. This is summarized in Exhibit 3.

| Exhibit 3: Firm Value using the Adjusted Present Value Constant Debt/Equity | | | |
|---|---|---|---|
| Vu | 2,500.00 | E | 1,687.50 |
| | | | |
| TS | 187.50 | D | 1,000.00 |
| | | | |
| V | 2,687.50 | V | 2,687.50 |

### 3.2 Weighted Average Cost of Capital

In order to do the same valuation using the Weighted Average Cost of Capital (WACC), we again need to find the right way to *lever* the cost of equity. In order to derive this cost of equity, analogous to Section 2.2, the starting point is that the left hand side and the right hand side of the balance sheet as in Exhibit 3, must give the same dollar (required) return:

$$k_U \times V_U + k_U \times TS = k_E \times E + k_D \times D.$$

The key is now that the tax shield ($TS$) is based on $k_U$ as a discount factor, instead of $k_D$. Manipulating this equality, and using that the tax shield is given in Equation (9), the resulting cost of equity is

$$k_E = k_U + \frac{D}{E}\left(k_U - k_D\right),$$

which is equal to Equation (2). The difference with Equation (1), which results in case of a constant level of debt, is that the Debt/Equity ratio ($D/E$) is not taken after tax, but at gross value. This is not to say that we ignore taxes, but it is simply a result of the fact that the tax shield is now more risky: it reflects the risk of the unlevered firm rather than the risk of the debt. It is now this cost of equity that should be used in calculating the WACC as in Equation (7).

In our example we have for the cost of equity and the WACC:

$$\begin{aligned} k_E &= 8.0\% + \frac{1,000}{1,687.5} \times (8.0\% - 5.0\%) \\ &= 9.8\%, \\ WACC &= \frac{1,687.5}{2,687.5} \times 9.8\% + \frac{1,000}{2,687.5} \times 70\% \times 5.0\% \\ &= 7.4\%. \end{aligned}$$

Applying this WACC to the Free Cash Flow of 200, the resulting firm value is 200/7.4% = 2,687.5 as in the Adjusted Present Value calculation in Exhibit 3.

Again, we can rewrite the WACC by substituting (2) into (7) and obtain:

$$WACC = k_U - \frac{D \times T_C}{V} k_D. \tag{11}$$

Unlike Equation (8), in this setting the WACC does depend on $k_D$ as well as on the other terms that affect the WACC in (8).

## 3.3 Cash Flow to Equity

Again, the third valuation method is to value equity directly, by discounting the cash flow that accrues to the shareholders, the Cash Flow to Equity (CFE) and discouning it at the cost of equity. Since we are working in a framework withouth growth, the expected change in debt, $\triangle D$, equals zero, and the Cash Flow to Equity is the same as in Section 2.3. We can then value the equity by discounting the CFE at the (levered) cost of equity in (2), which euqals 9.8%, that is also present in the WACC.

As before, in our example, the Cash Flow to Equity is

$$CFE = 200 - 5\% \times 70\% \times 1000 = 165.$$

Discounting this at the cost of equity gives 165/9.8% = 1,687.5 as in Exhibit 3. This again illustrates that in our simple example the three valuation approaches indeed lead to the same value of equity as well as total firm value.

# 4 Conclusions

In Part 1 of this article we have shown that the APV, WACC and CFE methods for valuing companies lead to the same result, provided that the cost of equity and the tax shield are adjusted in the right way. The key element in valuing the tax shield or calculating the cost of equity is the financing policy. If the financing policy is to have a constant level of debt, then the tax shield is calculated by discounting the tax savings at the cost of debt ($k_D$) and by using Equation (1) for (un)levering the cost of equity. If the financing policiy is to have constant Debt/Equity ratio ($D/E$), then the tax shield is calculated by discounting the tax savings at the unlevered cost of capital ($k_U$) and by using Equation (2) for (un)levering the cost of equity. The cost of equity according to Equations (1) and (2) are also the relevant inputs for the WACC and the CFE under the respective financing policies.

It is important to note that the equations given here only reflect the risk adjustments of the unlevered cost of capital ($k_U$), the levered cost of equity ($k_E$) and the WACC. They can be used easily in case the Free Cash Flow is expected to be constant (i.e., no growth) and are useful primarily for calculating the terminal or continuing value of a company. In case there is growth, additional adjustments need to be made in the three valuation methods. This issue will be addressed in Part II of this article.

# References


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